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FREQUENCY RANGES FOR EXISTENCE OF WAVES IN A COLD, COLLISIONLESS HYDROGEN PLASMA

by Richard R. Woollett

Lewis Research Center
Cleveland, Ohio

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SUMMARY

The dispersion relation of an electromagnetic wave propagating in a cold collisionless plasma at an arbitrary angle with the magnetic field was solved for an atomic hydrogen plasma. The results indicate that wave propagation is not possible for all values of frequency if the density, magnetic-field strength, and axial wavelength are held constant. Those regions of the frequency spectrum where wave propagation is possible are referred to as existence regions. Within these regions, bounded, cold collisionless plasmas exhibit natural modes of oscillation. Consequently, if experimental data fall in an existence region, the model of a cold collisionless plasma might be used to describe the phenomenon. The existence regions are determined for a range of magnetic fields from 10^2 to 10^5 gauss, a range of ion densities from 10^{10} to 10^{14} ions per cubic centimeter, and a range of selected axial wavelengths from 10 to 100 centimeters. The computations are compared with those resulting from an approximate dispersion relation that neglects the effects of electron inertia.

INTRODUCTION

The results of experiments concerning wave propagation in plasmas may exhibit behavior that cannot be predicted. Such results may possibly be interpreted as a consequence of wave propagation if they occur in a frequency range (existence region) obtained from the solution of the dispersion relation. This relation, obtained from the wave equation, depends on the model chosen to represent the experimental plasma.

A plasma model that has been successful in predicting many of the important types of wave motion occurring in real plasmas is that of a uniform, cold collisionless plasma. For this model, the dispersion relation for the propagation of plane waves in an infinite medium is derived in reference 1. This dispersion relation has been solved for many different cases by many different investigators. Solutions for the atomic hydrogen plasma pertinent to this report are presented in reference 2. The analysis of reference 3 demonstrates that this dispersion relation applies also to the oscillations of a plasma cylinder bounded by either a vacuum, a dielectric wall, or a wall of infinite conductivity.

In this report, solutions of the dispersion relation for the cold collisionless plasma are examined to determine the regions of the frequency spectrum for which solutions are possible. If an observed phenomenon occurs within one of these existence regions, a more detailed study using the cold, collisionless plasma model is suggested. If it does not lie within an existence region, the phenomenon is either not compatible with this simple plasma model or not the result of a wave phenomenon at all.

Such regions of solution have been calculated previously for ion cyclotron waves (ref. 4). The frequencies associated with the various wave numbers were obtained in reference 4 from an approximate dispersion relation (ref. 5) that did not include the effects of the electron inertia term that appears in the generalized Ohm's law. The resulting expressions of references 4 and 5 are shown in reference 6 to be valid for frequencies not much above the ion cyclotron frequency for propagation angles not too near 90° from the direction of the magnetic field. The dispersion relation used herein, which was obtained by using the generalized Ohm's law of reference 7, includes the inertia term. The resulting expressions for plane waves are essentially equivalent to those developed in reference 1. With the use of this more general dispersion relation, expressions that are applicable for all frequencies can be obtained.

The lower and upper frequency limits, which bound the regions where steady-state solutions to the wave equations are possible, are calculated herein as a function of plasma density for an atomic hydrogen plasma. The results cover a range of magnetic-field strengths from 10^2 to 10^5 gauss, frequencies from 10^{-1} to 10^5 times the ion cyclotron frequency, ion densities from 10^{10} to 10^{14} ions per cubic centimeter, and axial wavelengths from 10 to 100 centimeters.

The cold, collisionless plasma model used in obtaining these results applies equally well to fully ionized gases or to plasma containing electrons, ions, and neutrals, since the neutrals behave only as a nonparticipating background gas.

SYMBOLS

B magnetic-field strength, gauss

c velocity of light, cm/sec

e electronic charge, esu

K nondimensionalized wave number, kc/ω_{ci}

k wave number, $2\pi/\lambda$, cm^{-1}

L lower limit on existence region

m_e mass of electron, g

m_i mass of hydrogen ion, g

n ion density, ions/cm^3

n_e electron density, electrons/cm^3

U upper limit on existence region

λ wavelength, cm

Ω nondimensional angular frequency, ω/ω_{ci}

Ω_e ω_{ce}/ω_{ci}

Ω_p ω_p/ω_{ci}

ω angular frequency, radians/sec

ω_{ce} electron cyclotron frequency, $Be/m_e c$, radians/sec

ω_{ci} ion cyclotron frequency, $Be/m_i c$, radians/sec

ω_p plasma frequency, $4\pi n_e e^2/m_e$, radians/sec

Subscripts:

z coordinate parallel to magnetic-field direction

$\perp$ coordinate perpendicular to magnetic-field direction, rectangular or cylindrical coordinates

ANALYSIS

The propagation of an electromagnetic wave in a plasma at an arbitrary angle with the magnetic field (oblique propagation) depends on wave number, propagation angle, frequency, density, and field strength of the steady superimposed magnetic field. The first two variables, wave number and propagation angle, may be replaced by axial (k_z) and transverse ($k_\perp$) wave numbers, which are more conveniently related to experimental apparatus. The relation between the wave numbers and the various other parameters is obtained from the dispersion relation. For a given plasma and a given frequency of oscillation there can be an infinite and continuous set of combinations of k_z and $k_\perp$. The boundary conditions select a discrete (but infinite) set of k_z and $k_\perp$ from the total combinations. Frequently, the plasma of interest can be considered as being unbounded in certain directions. If the unbounded direction is along z, any k_z can be impressed upon the plasma by a device like a Stix coil (ref. 6). The actual value existing in a plasma can be measured. For such a discrete value of k_z it has been demonstrated (ref. 4) that, when the transverse wave number is permitted to vary over the range $0 \leq k_\perp < \infty$ (with density and field intensity fixed), the frequency of an undamped wave is confined to a limited range of permissible values. To determine the allowed frequencies, the dispersion relation can be expressed as a polynomial in Ω^2 with field strength, density, and axial wavelength treated as parameters.

The dispersion relation obtained from the simultaneous solution of the equation of motion of a fluid element (eq. 2-11, ref. 7), generalized Ohm's law (eq. 2-12, ref. 7), and Maxwell's equations can be expressed as

$$\begin{vmatrix} S - k_z^2 & -iD & k_z k_\perp \\ iD & S - (k_z^2 + k_\perp^2) & 0 \\ k_z k_\perp & 0 & P - k_\perp^2 \end{vmatrix} = 0 \tag{1}$$

where

$$S = \frac{\omega^2}{c^2}\left[1 - \frac{\omega_p^2(\omega^2 - \omega_{ce}\omega_{ci})}{(\omega^2 - \omega_{ce}\omega_{ci})^2 - \omega^2\omega_{ce}^2}\right] \tag{2a}$$

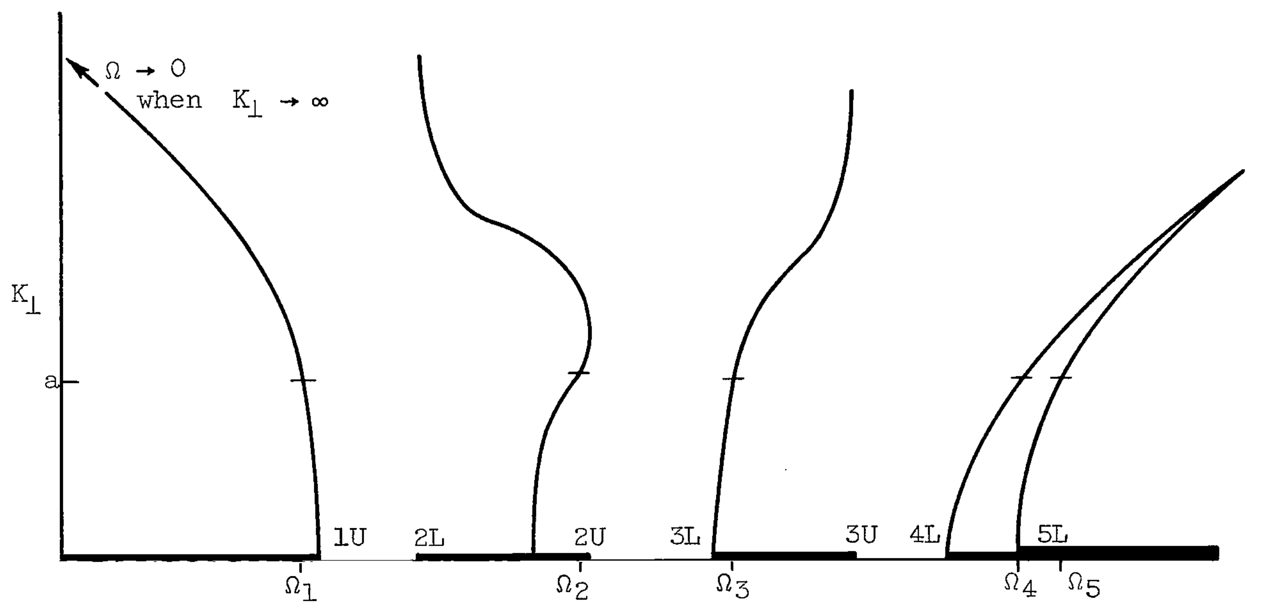
$$D = -\frac{\omega^3\omega_p^2\omega_{ce}}{c^2\left[(\omega^2 - \omega_{ce}\omega_{ci})^2 - \omega^2\omega_{ce}^2\right]} \tag{2b}$$

$$P = \frac{\omega^2}{c^2}\left(1 - \frac{\omega_p^2}{\omega^2}\right) \tag{2c}$$

In the derivation of equation (1) all effects attributable to collisions have been neglected. These would include terms involving pressure and resistivity (ref. 7). Moreover, expressions (1) and (2) are not identical to those given in reference 1 since terms of the order m_e/m_i compared with unity have been neglected in the present investigation. This, however, does not introduce a significant discrepancy between the two analyses (ref. 2). After some algebraic manipulation and nondimensionalization of the parameters, equation (1) reduces to

$$
\begin{aligned}
&-\Omega^{10} + \Omega^{8}\left[2\left(K_\perp^2 + K_z^2\right) + \Omega_e^2 + 3\Omega_p^2\right] - \Omega^{6}\left[\left(K_\perp^2 + K_z^2\right)^2 + 2\left(K_\perp^2 + K_z^2\right)\Omega_e^2\right. \\
&\quad \left. + 4\left(K_\perp^2 + K_z^2\right)\Omega_p^2 + \Omega_e^2 + 2\Omega_p^2\Omega_e + 3\Omega_p^4 + \Omega_e^2\Omega_p^2\right] + \Omega^{4}\left[\left(K_\perp^2 + K_z^2\right)^2\Omega_e^2\right. \\
&\quad + \left(K_\perp^2 + K_z^2\right)^2\Omega_p^2 + 2\left(K_\perp^2 + K_z^2\right)\Omega_e^2 + \left(3K_\perp^2 + 2K_z^2\right)\Omega_p^2\Omega_e + 2\left(K_\perp^2 + K_z^2\right)\Omega_p^4 \\
&\quad \left. + \left(K_\perp^2 + 2K_z^2\right)\Omega_e^2\Omega_p^2 + \Omega_e^2\Omega_p^2 + 2\Omega_p^4\Omega_e + \Omega_p^6\right] - \Omega^{2}\left[\left(K_\perp^2 + K_z^2\right)^2\Omega_e^2\right. \\
&\quad + K_\perp^2\left(K_\perp^2 + K_z^2\right)\Omega_p^2\Omega_e + K_z^2\left(K_\perp^2 + K_z^2\right)\Omega_e^2\Omega_p^2 + \left(K_\perp^2 + 2K_z^2\right)\Omega_e^2\Omega_p^2 \\
&\quad \left. + \left(K_\perp^2 + 2K_z^2\right)\Omega_p^4\Omega_e\right] + K_z^2\left(K_\perp^2 + K_z^2\right)\Omega_e^2\Omega_p^2 = 0 \qquad (3)
\end{aligned}
$$

As a particular parameter is varied, say $K_\perp$, the roots of equation (3) are grouped into bands as shown in the following sketch, which is a log log plot of $K_\perp$ with respect to Ω for a fixed K_z:



Roots of eq. (3), Ω

For example, the roots of equation (3) for the value $K_\perp = a$ are indicated on the curves. Such curves are obtained for each combination of values of λ_z, B, and n. The existence regions are the ranges of Ω for which values of $K_\perp$

exist. These existence regions are indicated by the heavy horizontal bars obtained by projecting the root curves on the abscissa. To identify the various regions, the lower and the upper limit of each are labeled L and U, respectively, and the first to the fifth regions are identified by numbers 1 to 5 preceding L and U. The frequency limits of each of these sets as a function of density, axial wavelength, and magnetic-field strengths are the quantities that are of most interest herein and, consequently, constitute the major part of the plots. The first region has been referred to as the ion cyclotron band (ref. 4). Since the second region depends on both electron and ion characteristics, it will be called the hybrid band.

Curves of $k_\perp$ as a function of Ω, such as that in the sketch, are presented in figure 1 for magnetic fields of 10^4 gauss, axial wavelengths of 10 and 40 centimeters, and ion densities of 10^{10} to 10^{14} ions per cubic centimeter. One characteristic of the dispersion relation depicted in figure 1 is that in four of the five existence regions $k_\perp$ is a monotonic function of Ω. In the first branch around the ion cyclotron frequency it is a decreasing function, while in the third, fourth, and fifth branches it is an increasing function. Expressions for the boundaries of the existence regions can be obtained by setting $K_\perp = 0$ or ∞ in equation (3). The calculation for $K_\perp = 0$ yields five frequencies, four of which correctly yield band limits. The remaining root, which is for the second existence region, does not correspond necessarily to either the upper or the lower limit of the second band. This can be seen in figure 1(b) by comparing the 10^{10}, 10^{12}, and 10^{14} ion per cubic centimeter density curves. In the second band the condition $K_\perp = 0$ represents a lower limit at an ion density of 10^{14} ions per cubic centimeter, and an upper limit at 10^{10} ions per cubic centimeter and does not represent any band limit at 10^{12} ions per cubic centimeter.

For the case of $K_\perp = \infty$, the finite roots are determined by

$$\Omega^4 - \Omega^2\left(\Omega_e^2 + \Omega_p^2\right) + \Omega_e^2 + \Omega_p^2\Omega_e = 0 \tag{4a}$$

$$\Omega^2 = 0 \tag{4b}$$

Equation (4a) has two roots, the smallest of which does not correspond necessarily to the limits of the second band (fig. 1). The roots of the dispersion relation in the second band do not form a monotonic function of $K_\perp$ for some values of n. Consequently, for the region the frequency limits cannot be determined from either $K_\perp = 0$ or $K_\perp = \infty$.

It is of interest to compare the existence band obtained from the generalized dispersion relation with that obtained from the approximate relation employed in reference 5 in a study of ion cyclotron waves. It can be shown that the equation of reference 5 reduces to

$$\Omega^6\Omega_e^2 - \Omega^4\left[\left(K_\perp^2 + 2K_z^2\right)\Omega_e^2 + \Omega_e^2 + 2\Omega_p^2\Omega_e + \Omega_p^4\right] + \Omega^2\left[K_z^2\left(K_\perp^2 + K_z^2\right)\Omega_e^2 + \left(K_\perp^2 + 2K_z^2\right)\Omega_e^2 + \left(K_\perp^2 + 2K_z^2\right)\Omega_p^2\Omega_e\right] - K_z^2\left(K_\perp^2 + K_z^2\right)\Omega_e^2 = 0 \tag{5}$$

This expression, being of the third degree in Ω^2, indicates that for a given $K_\perp$, K_z, and $\Omega_p(n)$ it is possible to generate waves of three different frequencies. Equation (3) is of the fifth degree in Ω^2 and yields five different frequencies. Two of the high-frequency modes are therefore lost when the electron inertia effect is neglected.

RESULTS

The limits of the existence regions determined by equation (3) are plotted as functions of density in figure 2 for field intensities from 10^2 to 10^5 gauss, ion densities from 10^{10} to 10^{14} ions per cubic centimeter, and axial wavelengths of 10, 20, 40, and 100 centimeters. These plots show that there are five existence bands for plasma wave propagation. The first band (numbering from the left) always has a lower frequency limit of zero; the upper limit is dependent on axial wavelength, but essentially independent of field strength for the range of parameters investigated. The fourth and fifth bands always have infinite upper frequency limits. In addition, the fourth band always overlaps the fifth, and occasionally the third band overlaps the fourth, a situation that decreases the number of nonexistence regions.

Figure 2(c-3) may be studied as an example of the existence regions for a particular set of parameters. It can be seen that, for a density of 10^{12} ions per cubic centimeter, wave propagation is possible for some value of frequency within the following bands: first band, $0 < \Omega < 0.95$; second band, $13 < \Omega < 38$; third band, $1.9\times10^2 < \Omega < 1.9\times10^3$; fourth band, $6\times10^2 < \Omega < \infty$; fifth band, $2\times10^3 < \Omega < \infty$. The particular form of presentation that appears in figure 2 is most useful when experimental power absorption is examined as a function of the forcing frequency. Thus, if the energy absorption of a plasma under a radio frequency coil peaks at a frequency that is not in an existence region, the absorption is not associated with excitation of a natural mode in a cold collisionless plasma.

It is interesting to compare the results of equation (3) with those obtained from the dispersion relation presented in reference 5 and represented by equation (5) in the present report. The existence regions determined from equation (5) are plotted in figure 3 for a field intensity of 10^4 gauss, a range of ion density from 10^{10} to 10^{14} ions per cubic centimeter, and axial wavelengths of 10, 40, and 100 centimeters. Since $K_\perp$ is a monotonically increasing function of Ω in each of the three existence bands determined from equation (5), the lower and the upper limits for each region were determined by substituting the relation $K_\perp = 0$ and ∞, respectively, into the dispersion relation (eq. (5)). In addition to the loss of the two upper existence bands, the character of the ion cyclotron band changes. This band no longer extends to zero on the low-frequency side but has a definite nonzero lower boundary. At low densities the lower limit of the second band shifts to greater frequencies than those given by the five-root expression. As an example, at a density of 10^{10} ions per cubic centimeter and an axial wavelength of 40 centimeters, the lower limit of the second band obtained from the five-root expression is $1.7\ \omega_{ci}$, while the three-root equation indicates all the solutions of the second band are grouped around $50\ \omega_{ci}$. Agreement between the two models is found for the upper limit of the first band and the lower limit of the second band at high density.

There are graphical presentations, other than those of figure 2, that may be more convenient for use with certain types of plasma experiments. As an example, consider the case when Ω is varied by changing the magnetic field while maintaining the dimensional frequency ω at a fixed value. For this particular case it is more convenient to treat the magnetic-field intensity as a variable and the exciting frequency as a parameter. Results displayed in this manner can be obtained by cross plotting the data presented in figure 2. As an example, the existence regions at a λ_z of 40 centimeters and an ω of 9.6×10^7 per second are plotted in figure 4 as a function of magnetic field intensity.

CONCLUDING REMARKS

Because of the particular nature of the dispersion relation for the propagation of an electromagnetic wave of a given axial wavelength through a plasma, the total possibilities for wave propagation lie in restricted frequency bands. These density-dependent existence regions for a cold, collisionless, atomic hydrogen plasma are plotted for selected axial wavelengths from 10 to 100 centimeters, magnetic fields from 10^2 to 10^5 gauss, densities from 10^{10} to 10^{14} ions per cubic centimeters, and frequencies from 10^{-1} to 10^5 times the ion cyclotron frequency.

If an observed wave phenomenon occurs within one of these existence regions, a more detailed calculation of the cold collisionless plasma dispersion relation subject to the proper boundary conditions is indicated. If the phenomenon does not lie within an existence region, it is not compatible with this plasma model, and a more refined model should be considered. Such an analysis could possibly modify the existence regions or even introduce entirely new ones if additional plasma modes developed.

Lewis Research Center
National Aeronautics and Space Administration
Cleveland, Ohio, December 28, 1963

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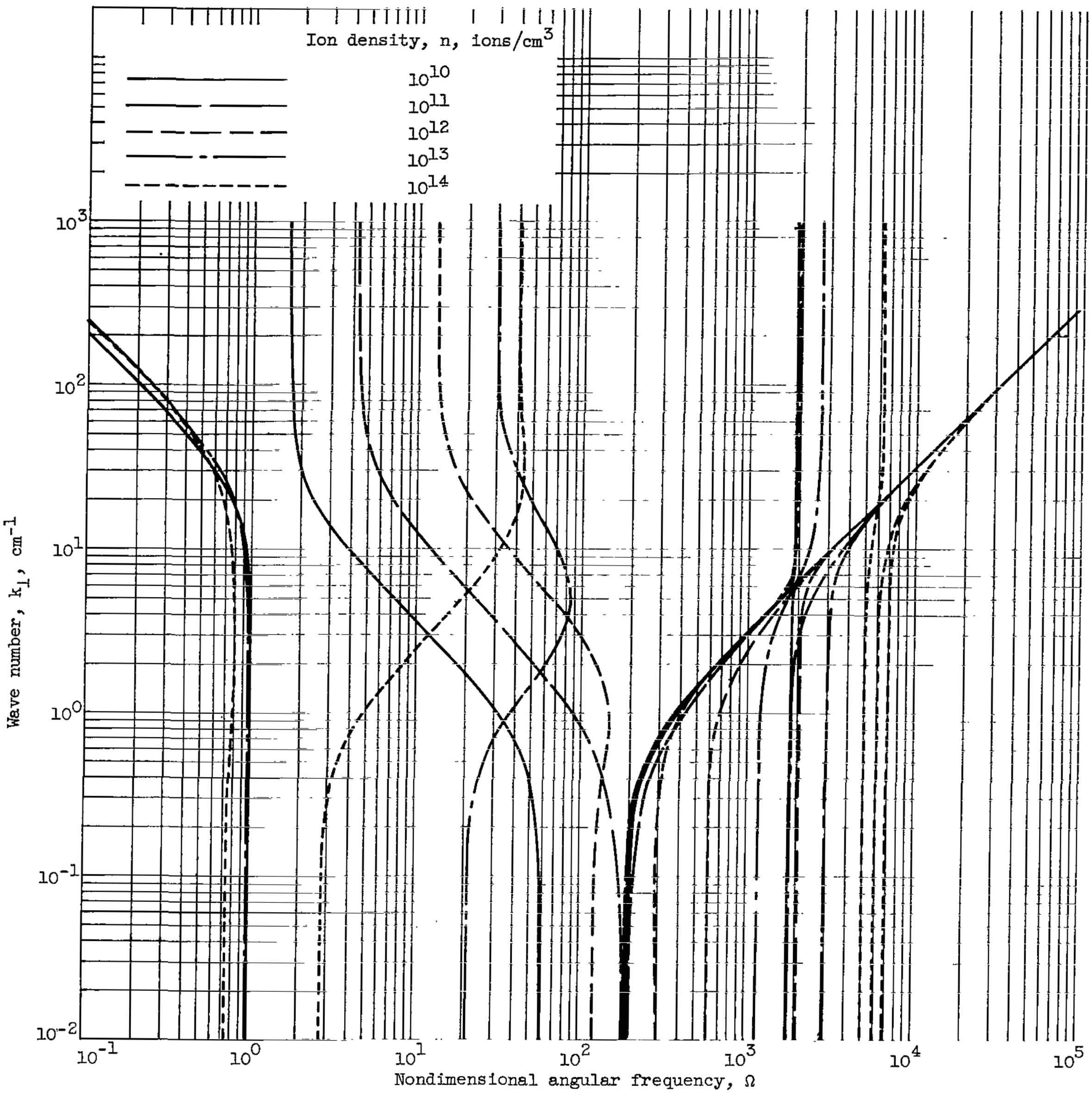
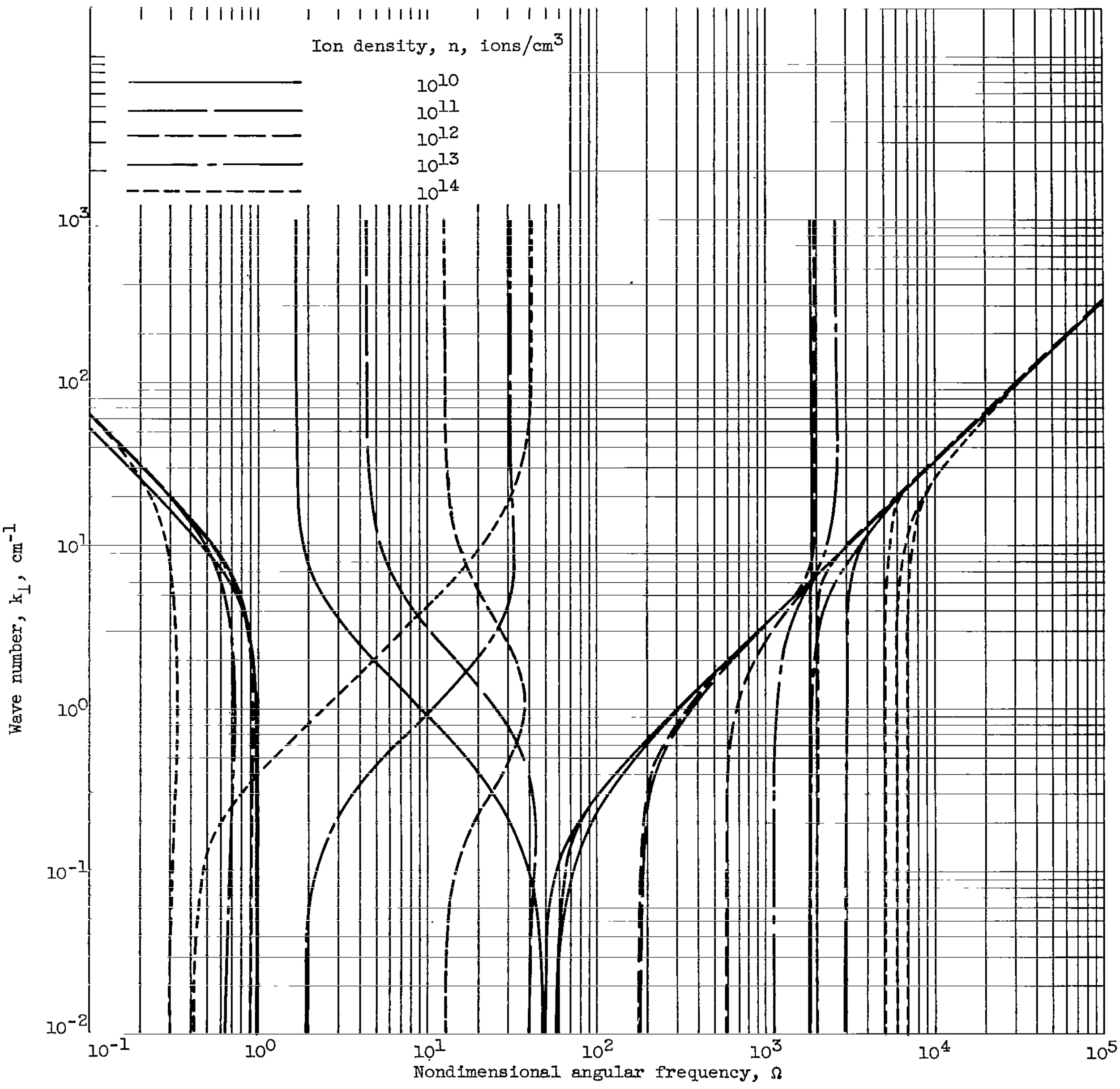


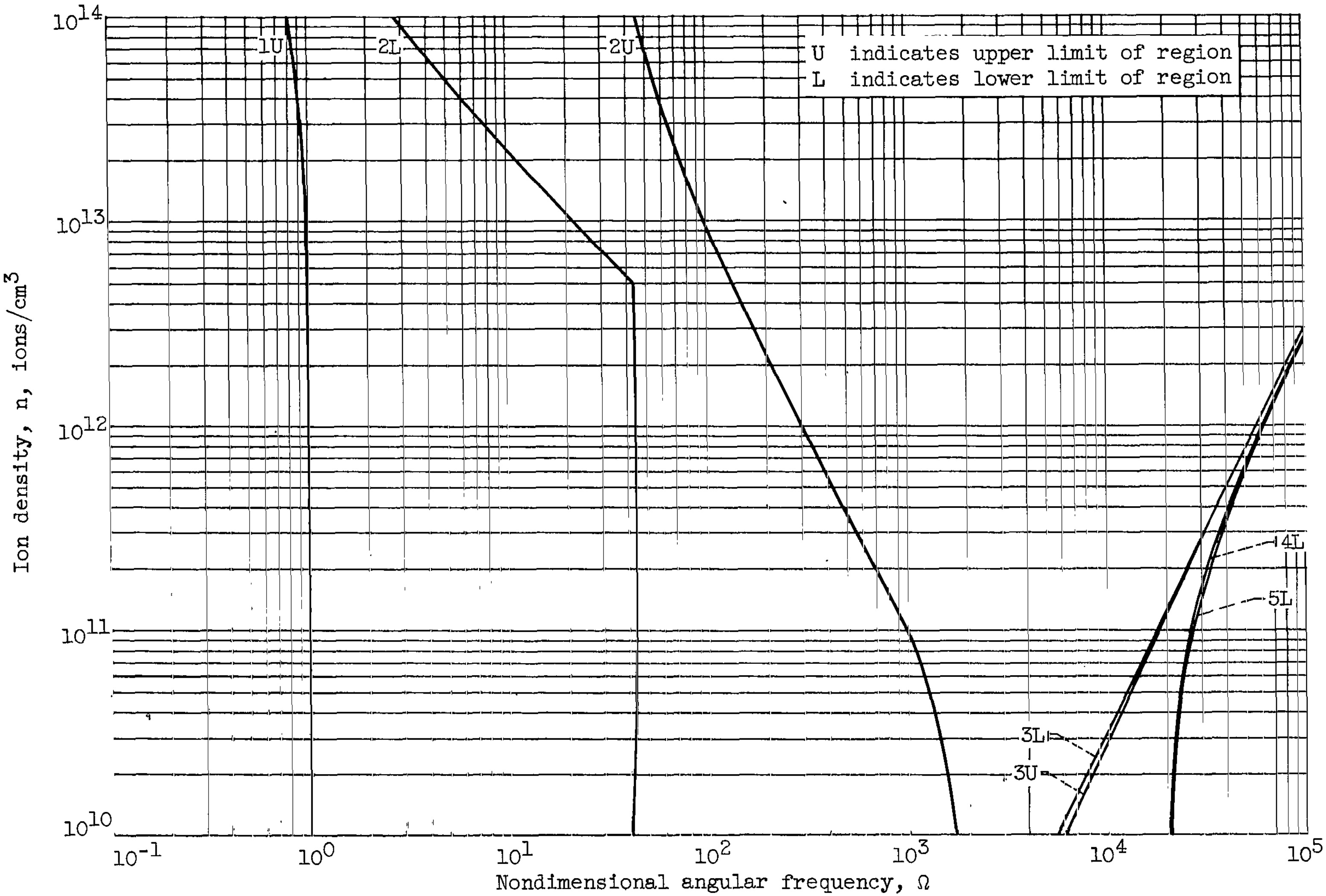
(a) Axial wavelength, 10 centimeters.

Figure 1. - Dispersion relation for hydrogen plasma. Magnetic field, 10^4 gauss.



(b) Axial wavelength, 40 centimeters.

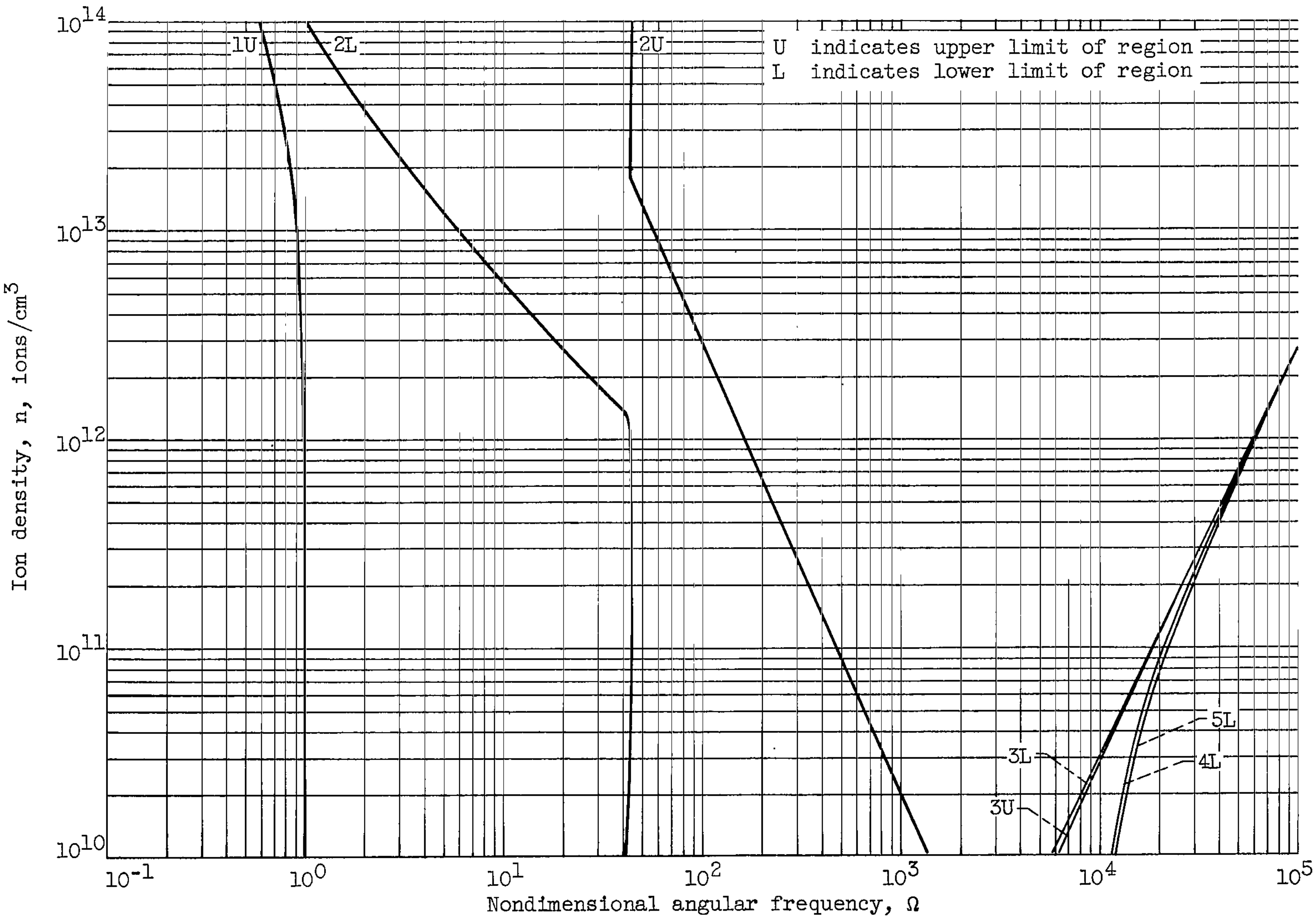
Figure 1. - Concluded. Dispersion relation for hydrogen plasma. Magnetic field, 10^4 gauss.



(a-1) Axial wavelength, 10 centimeters.

(a) Magnetic field, 10^2 gauss.

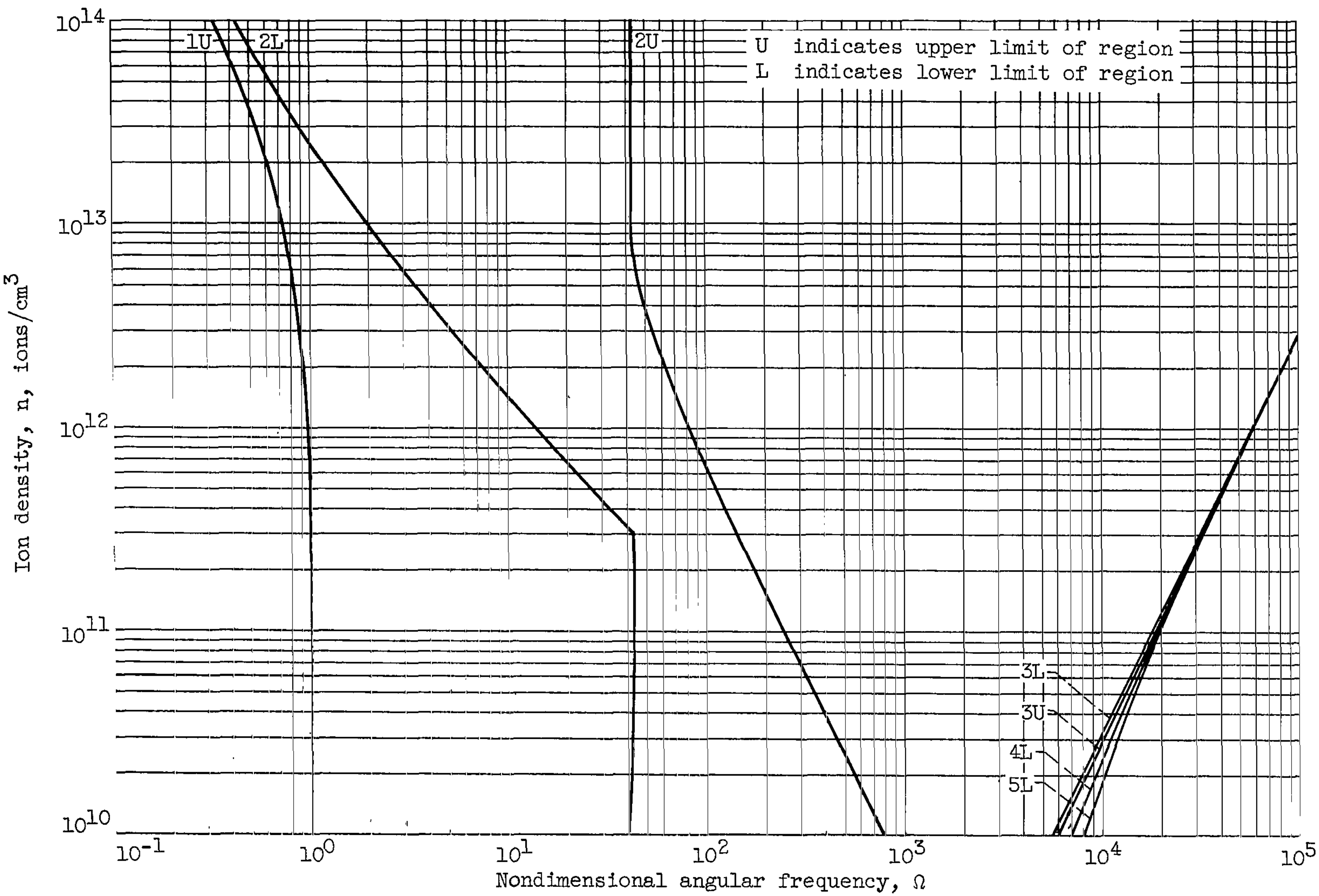
Figure 2. - Existence regions for hydrogen plasma waves.



(a-2) Axial wavelength, 20 centimeters.

(a) Continued. Magnetic field, 10^2 gauss.

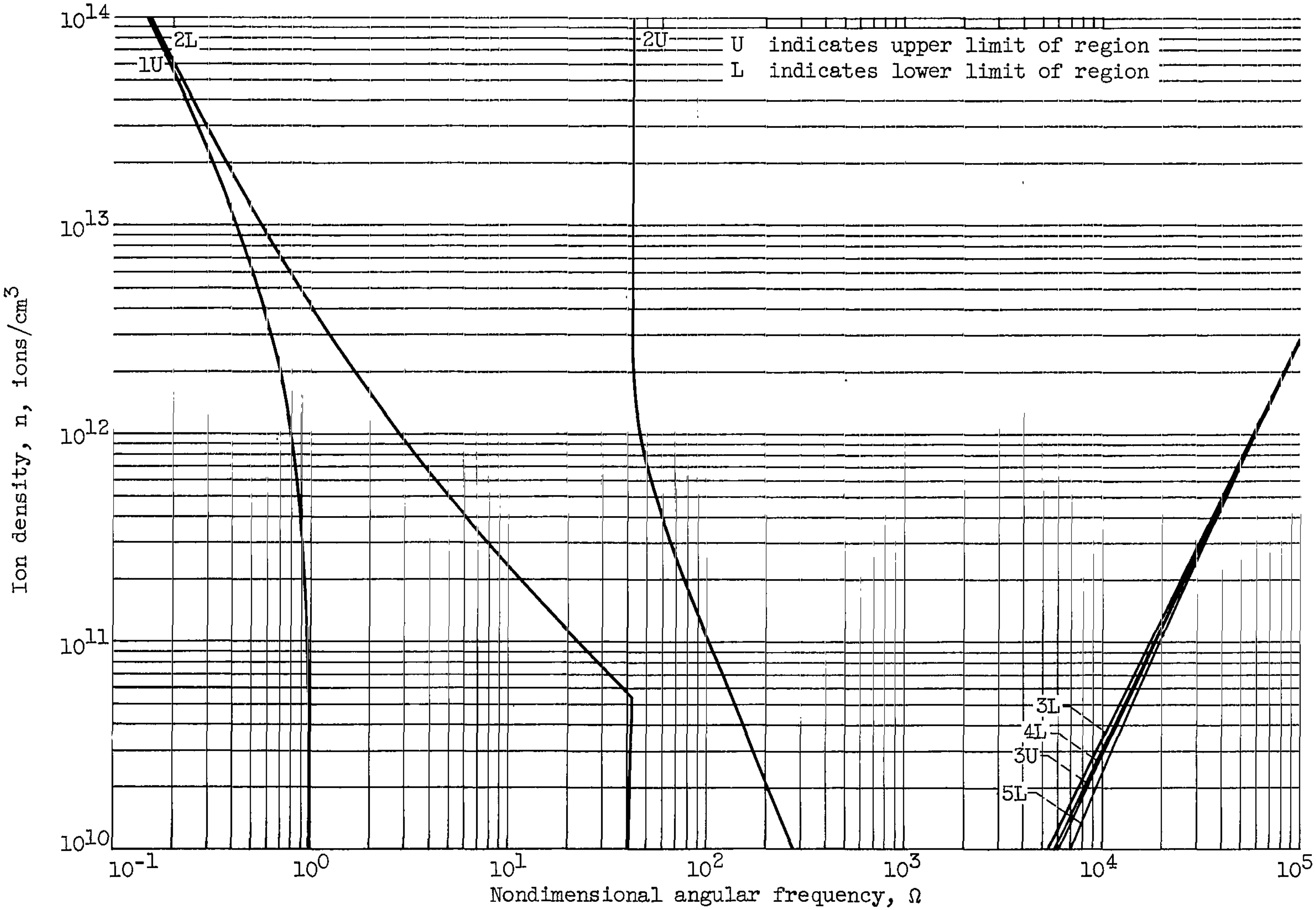
Figure 2. - Continued. Existence regions for hydrogen plasma.



(a-3) Axial wavelength, 40 centimeters.

(a) Continued. Magnetic field, 10^2 gauss.

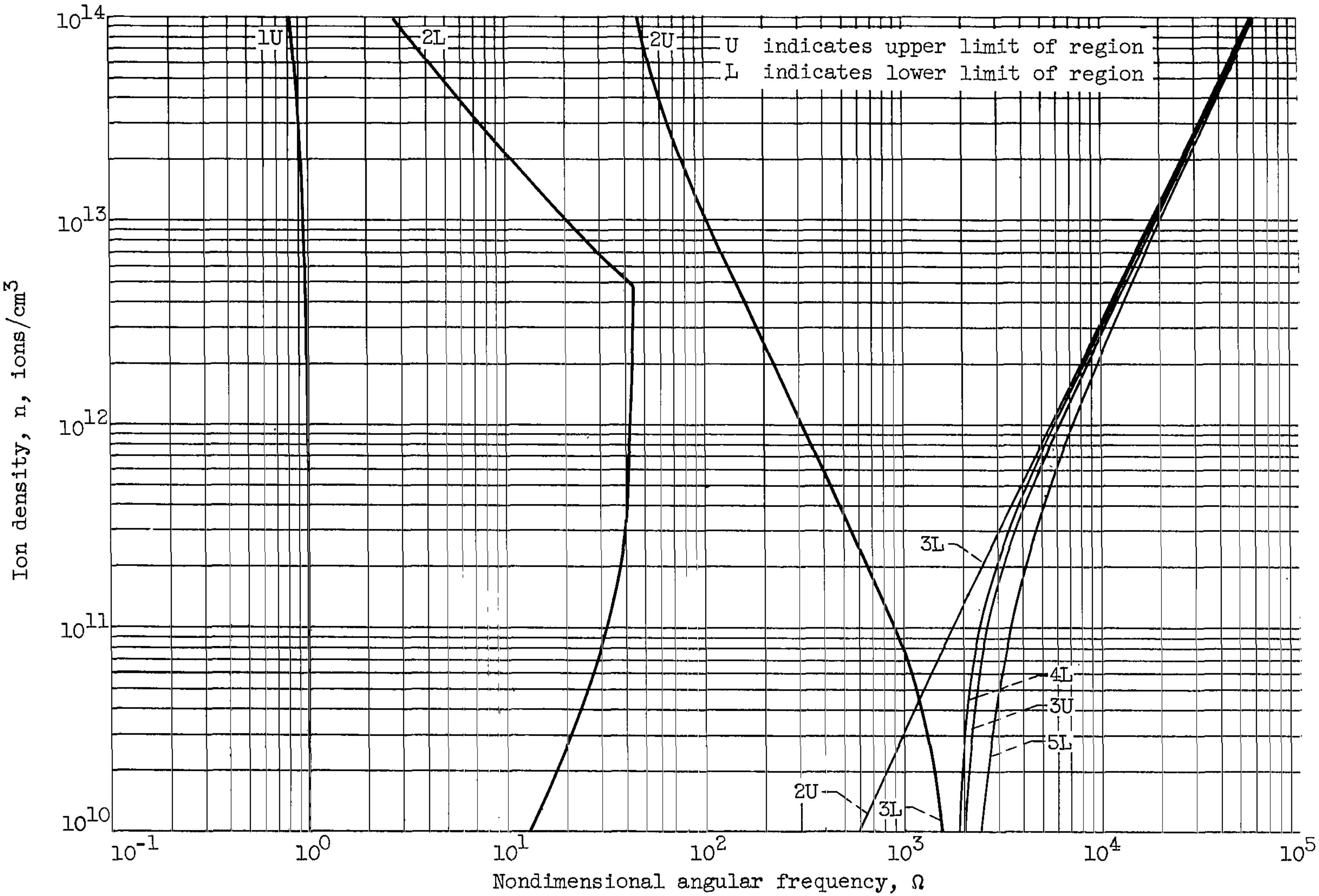
Figure 2. - Continued. Existence regions for hydrogen plasma waves.



(a-4) Axial wavelength, 100 centimeters.

(a) Concluded. Magnetic field, 10^2 gauss.

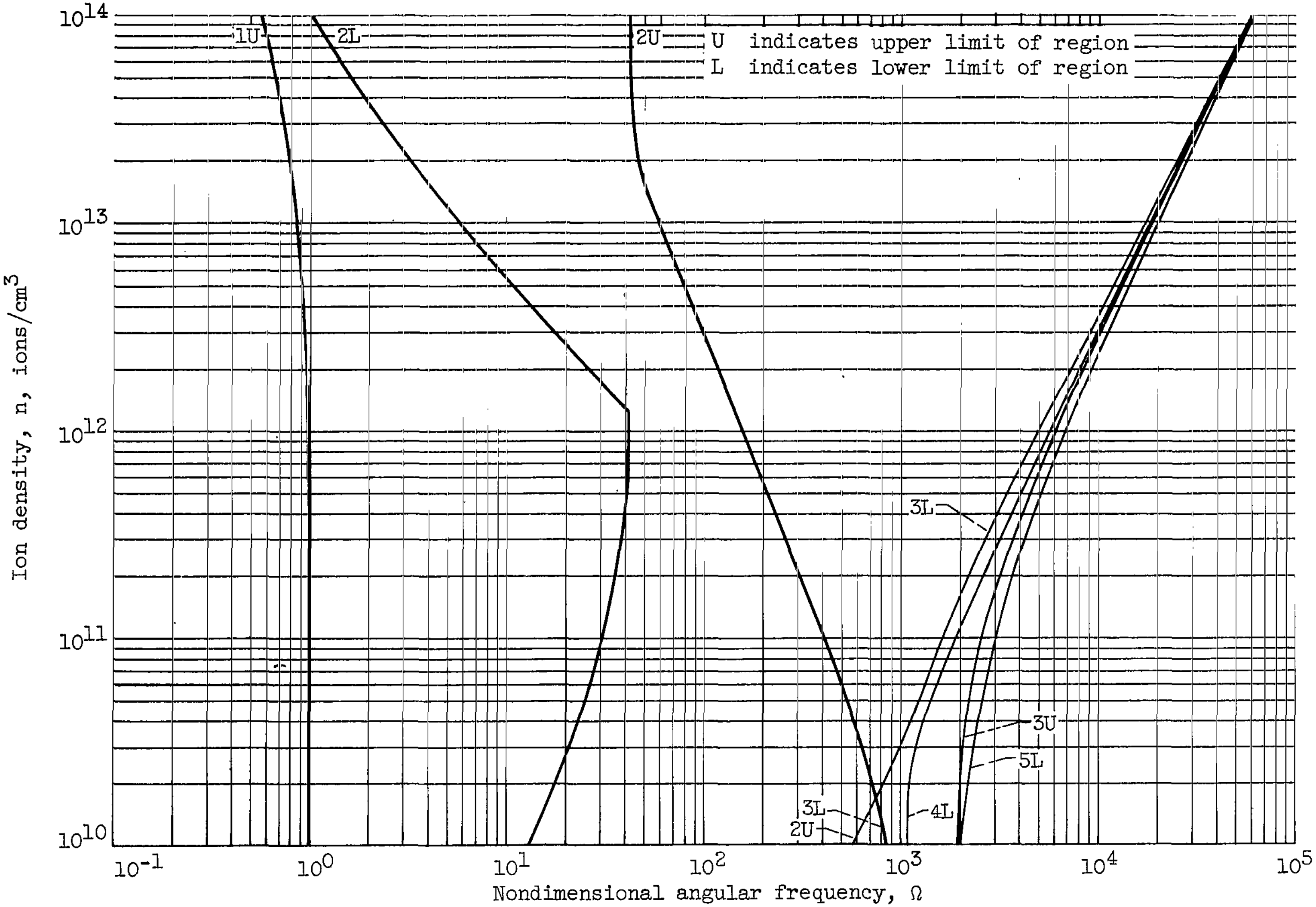
Figure 2. - Continued. Existence regions for hydrogen plasma.



(b-1) Axial wavelength, 10 centimeters.

(b) Magnetic field, 10^3 gauss.

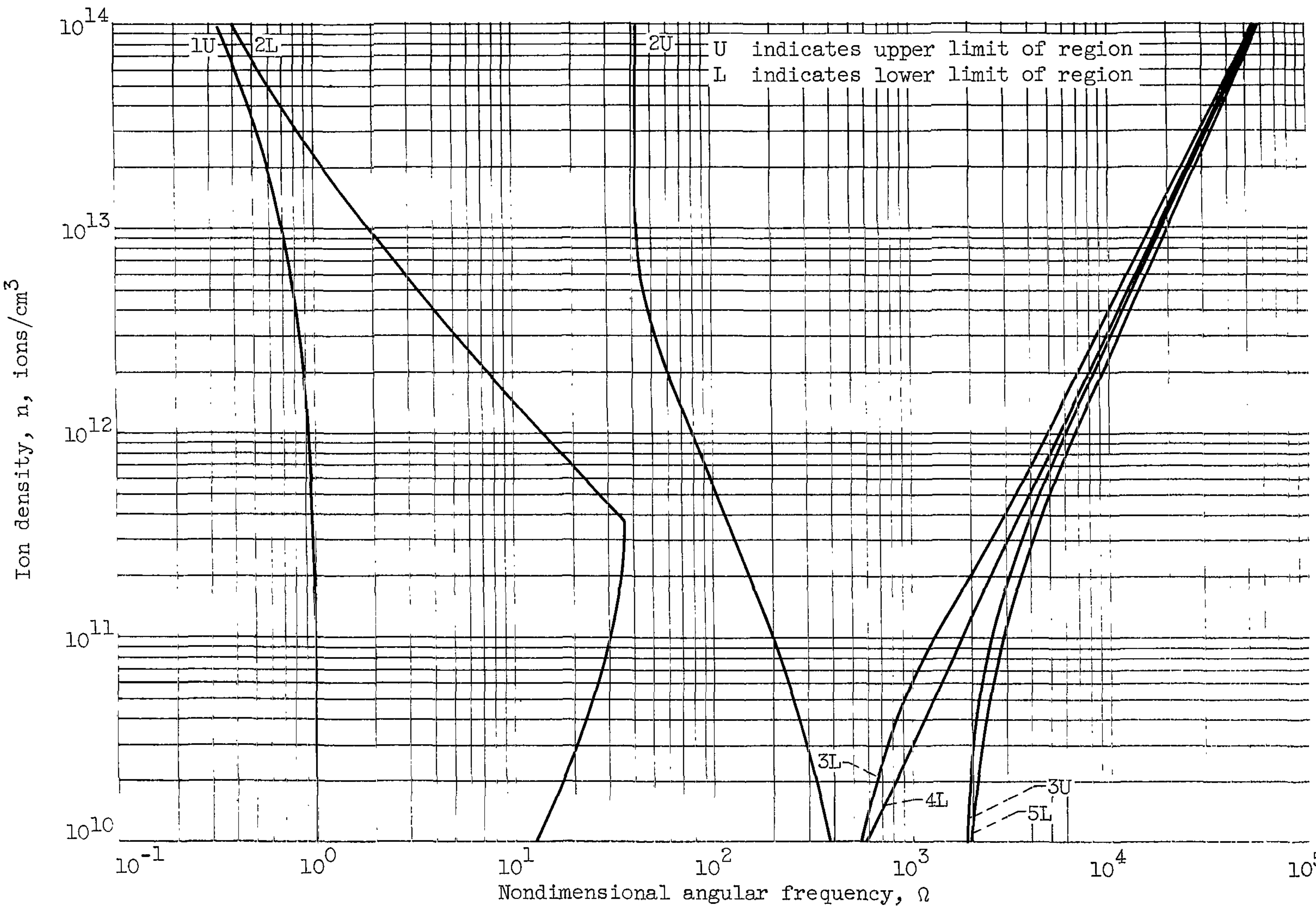
Figure 2. - Continued. Existence regions for hydrogen plasma waves.



(b-2) Axial wavelength, 20 centimeters.

(b) Continued. Magnetic field, 10^3 gauss.

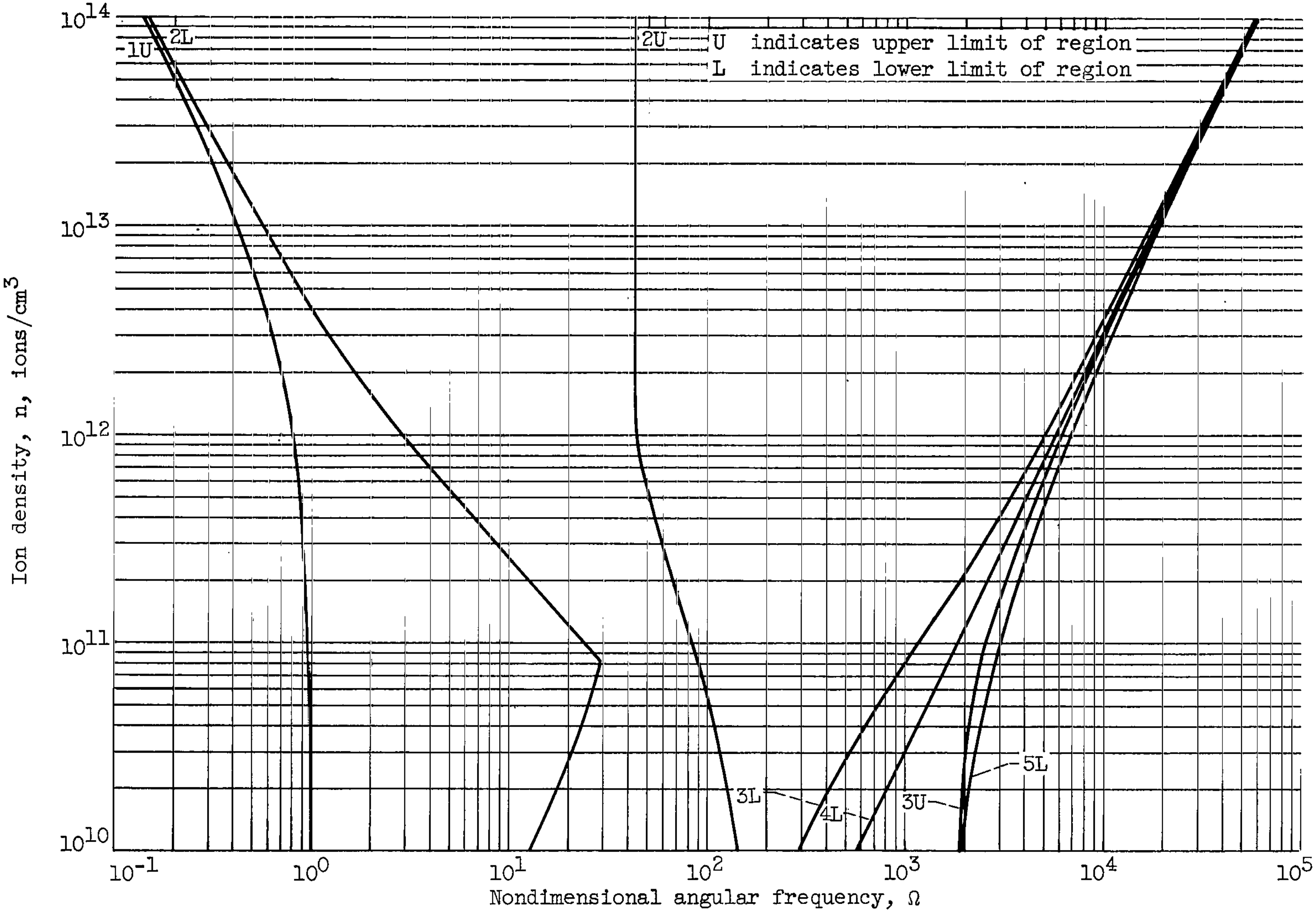
Figure 2. - Continued. Existence regions for hydrogen plasma waves.



(b-3) Axial wavelength, 40 centimeters.

(b) Continued. Magnetic field, 10^3 gauss.

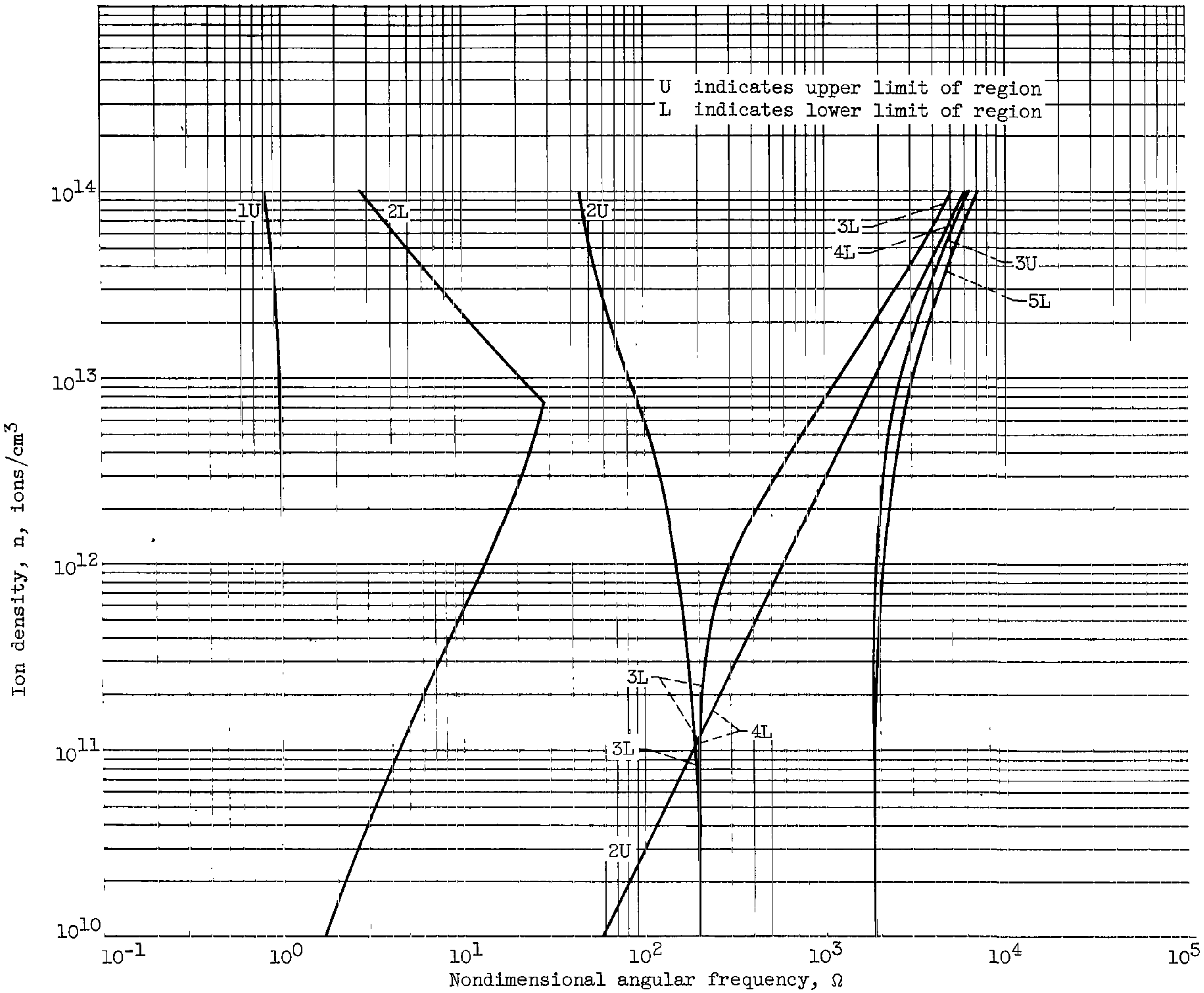
Figure 2. - Continued. Existence regions for hydrogen plasma waves.



(b-4) Axial wavelength, 100 centimeters.

(b) Concluded. Magnetic field, 10^3 gauss.

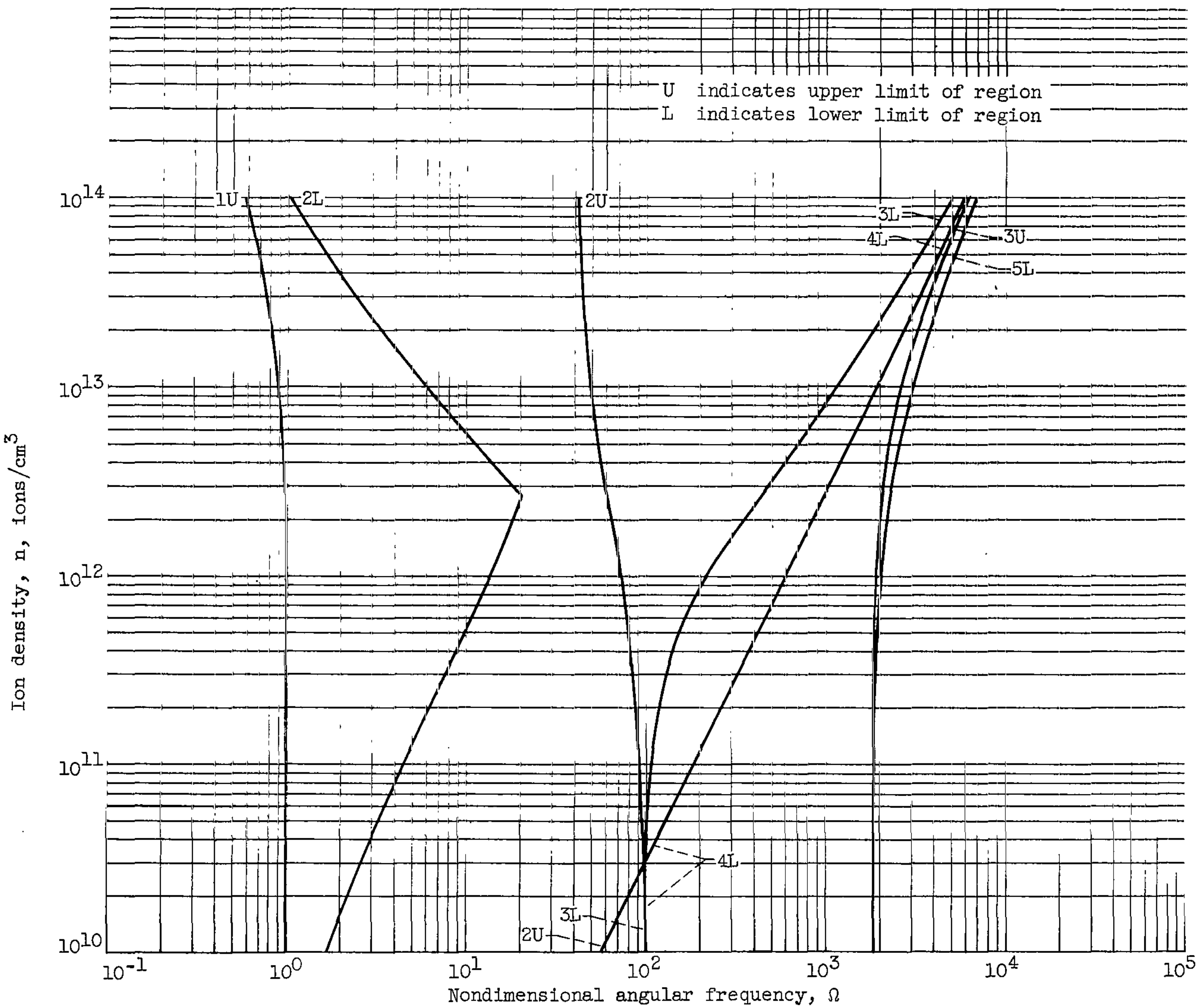
Figure 2. - Continued. Existence regions for hydrogen plasma waves.



(c-1) Axial wavelength, 10 centimeters.

(c) Magnetic field, 10^4 gauss.

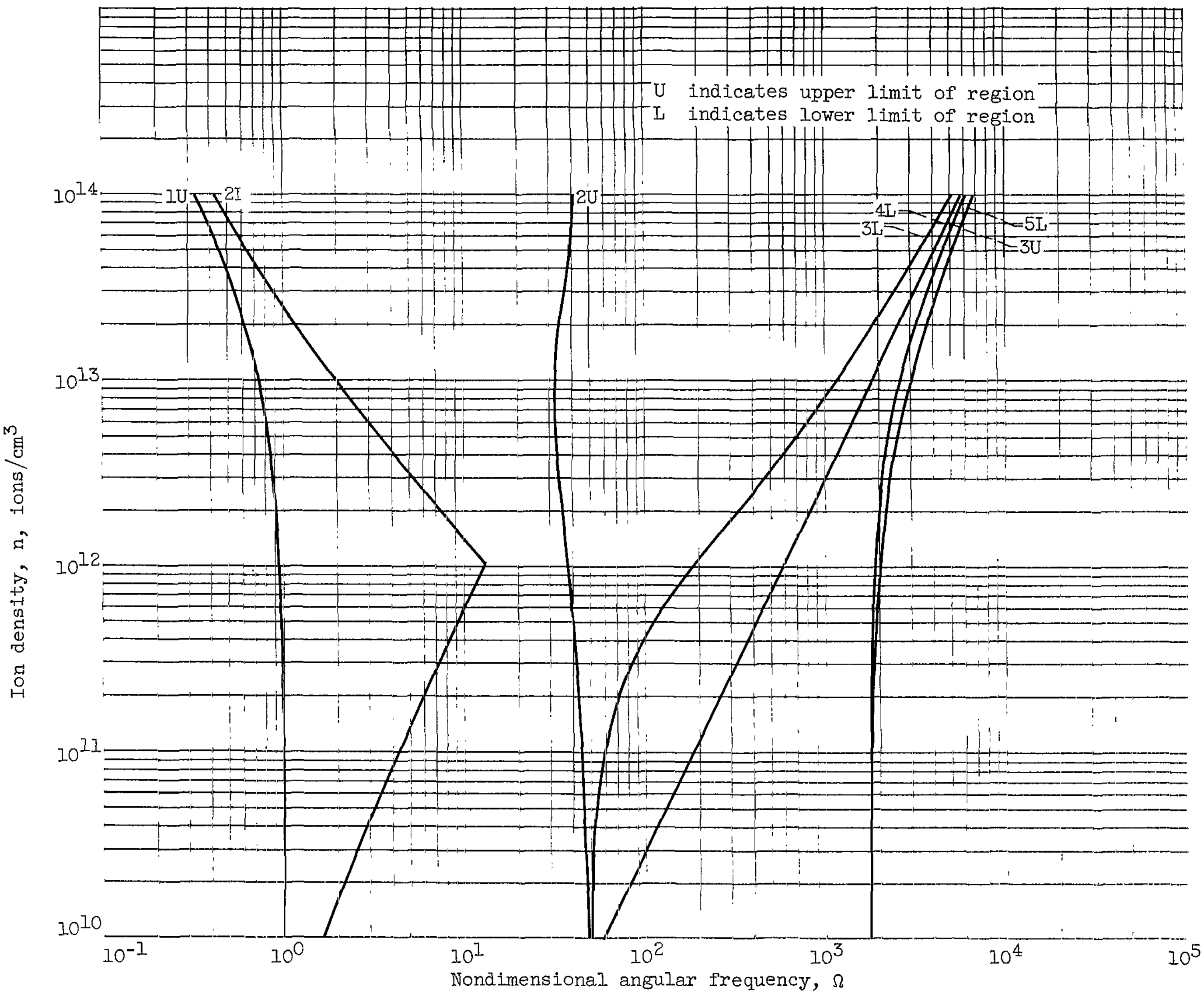
Figure 2. - Continued. Existence regions for hydrogen plasma waves.



(c-2) Axial wavelength, 20 centimeters.

(c) Continued. Magnetic field, 10^4 gauss.

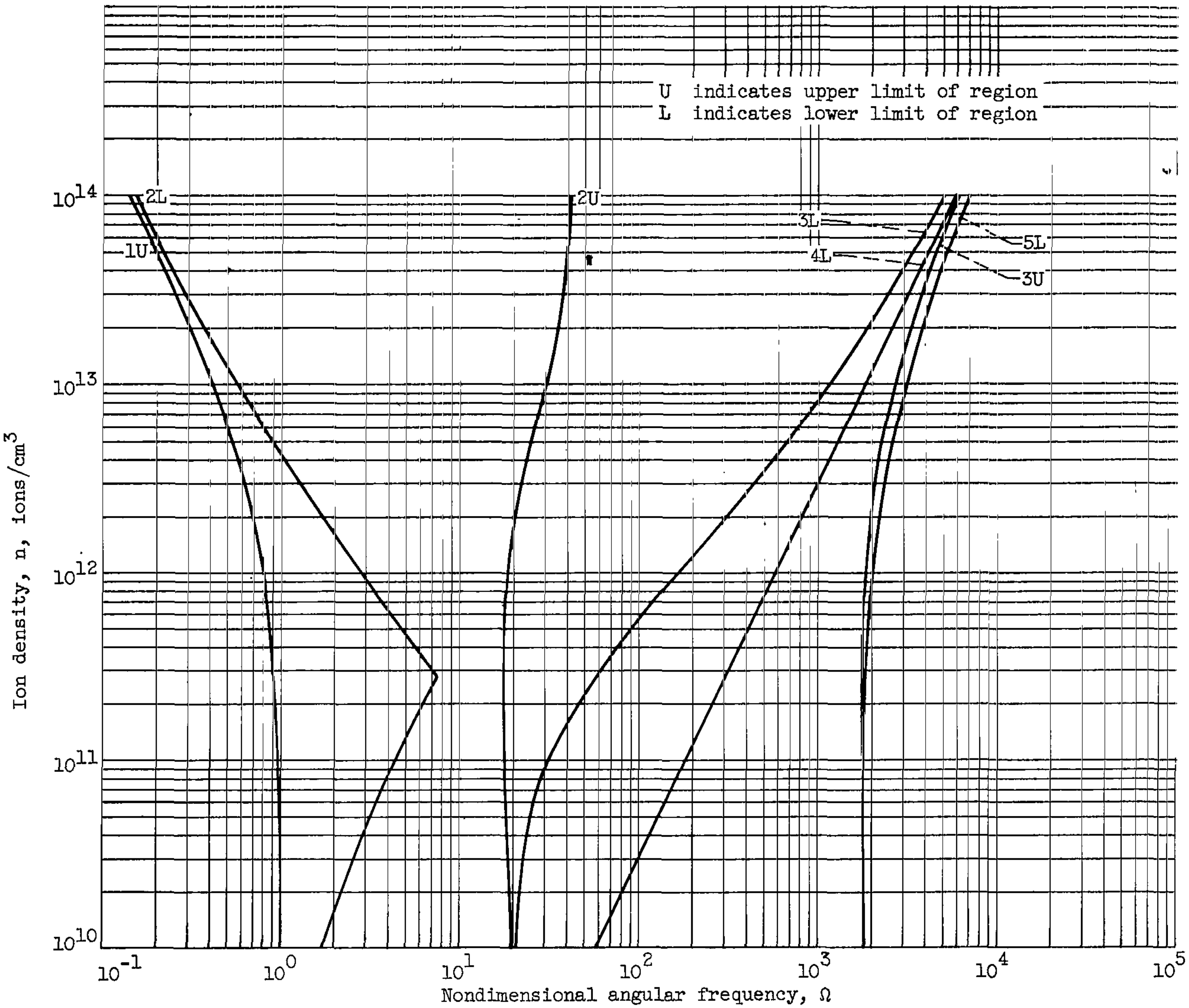
Figure 2. - Continued. Existence regions for hydrogen plasma waves.



(c-3) Axial wavelength, 40 centimeters.

(c) Continued. Magnetic field, 10^4 gauss.

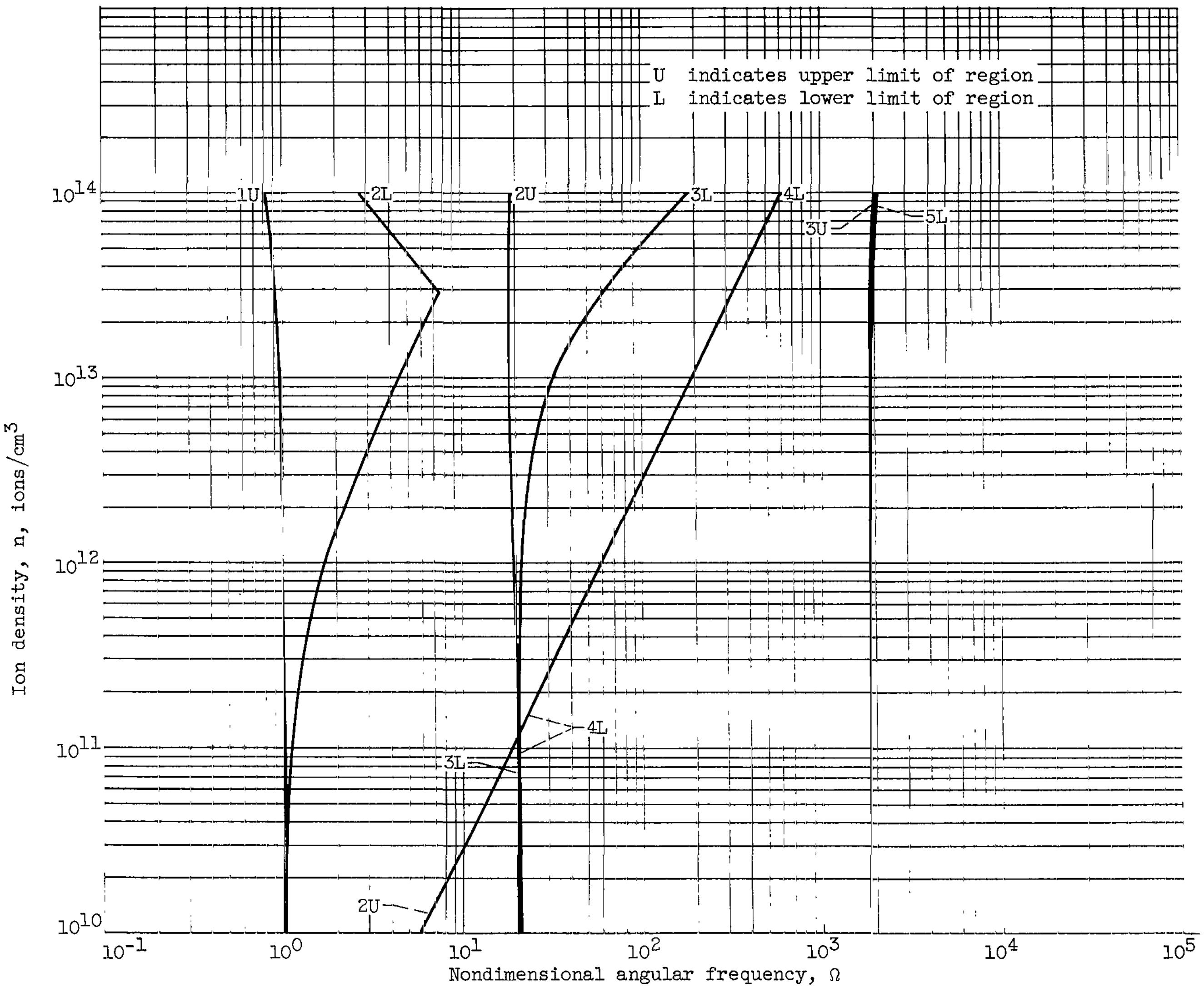
Figure 2. - Continued. Existence regions for hydrogen plasma waves.



(c-4) Axial wavelength, 100 centimeters.

(c) Concluded. Magnetic field, 10^4 gauss.

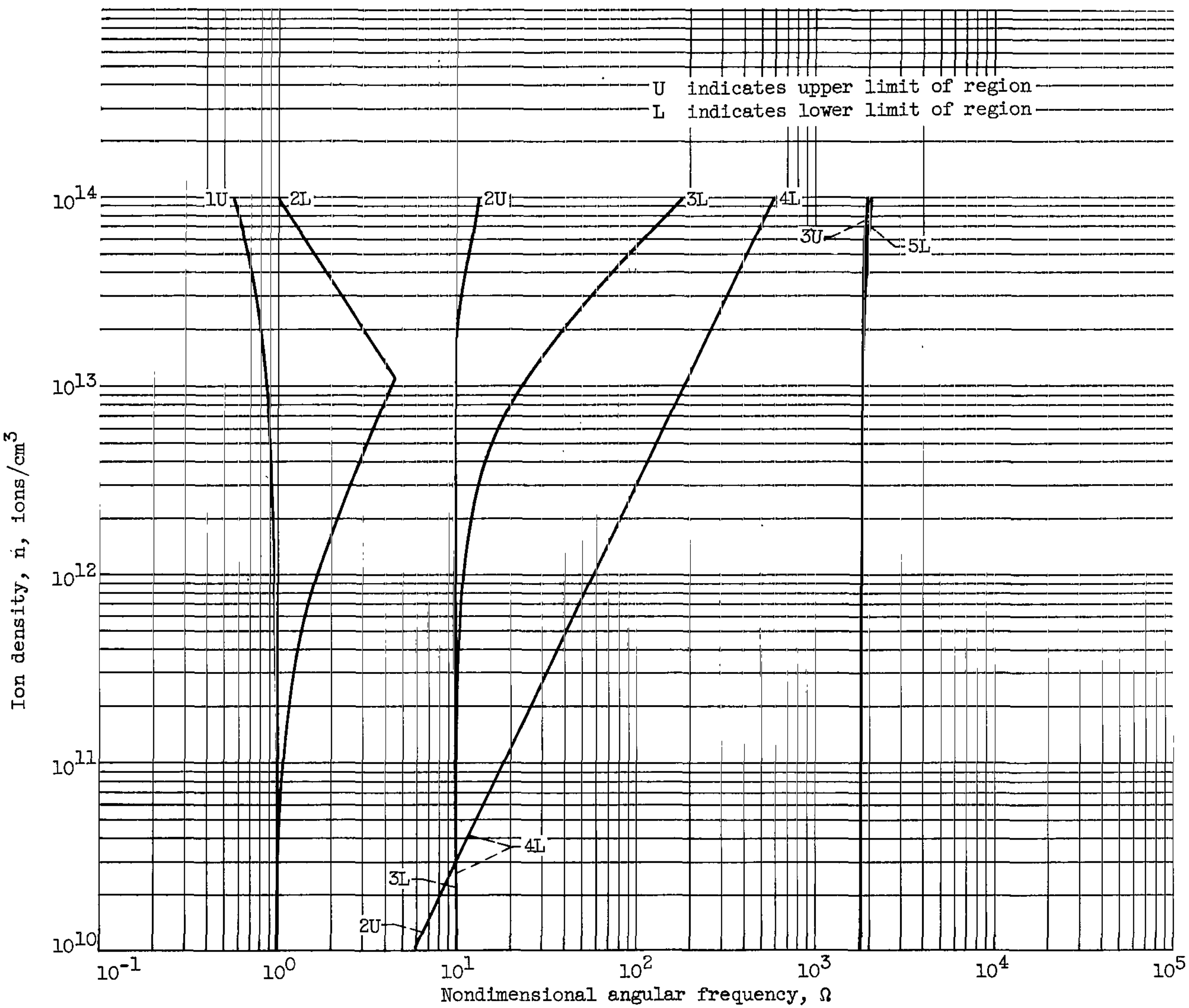
Figure 2. - Continued. Existence regions for hydrogen plasma waves.



(d-1) Axial wavelength, 10 centimeters.

(d) Magnetic field, 10^5 gauss.

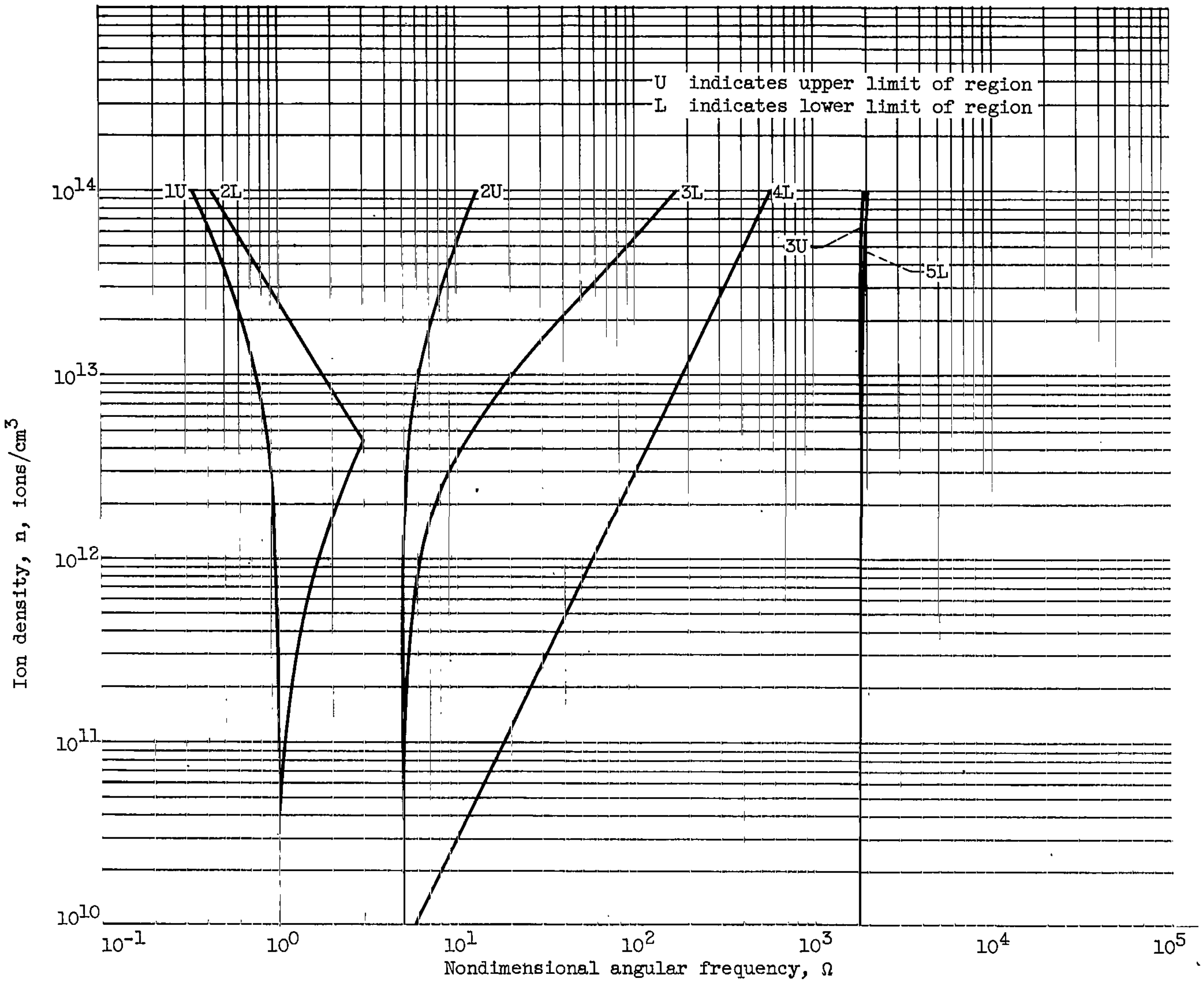
Figure 2. - Continued. Existence regions for hydrogen plasma waves.



(d-2) Axial wavelength, 20 centimeters.

(d) Continued. Magnetic field, 10^5 gauss.

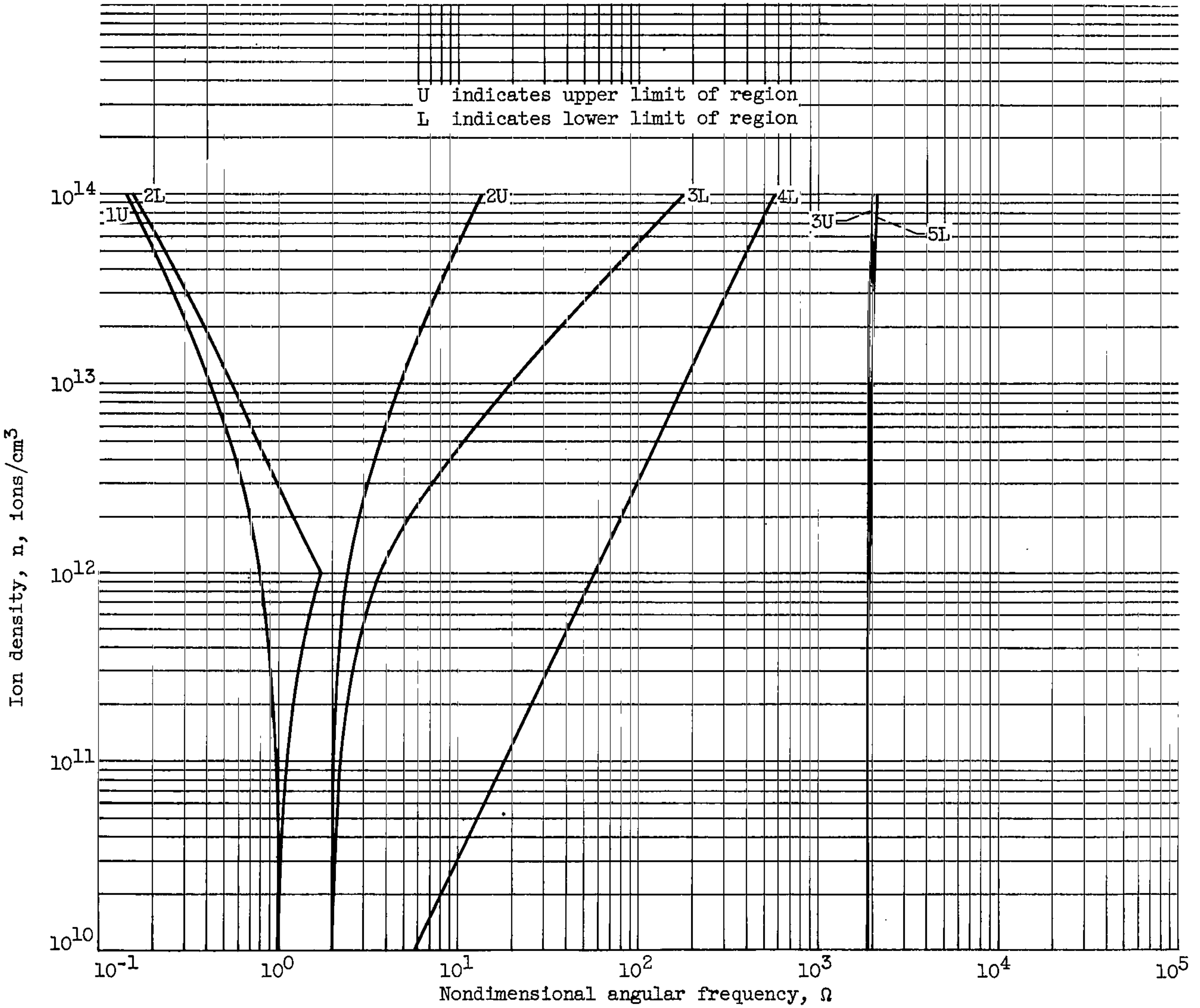
Figure 2. - Continued. Existence regions for hydrogen plasma waves.



(d-3) Axial wavelength, 40 centimeters.

(d) Continued. Magnetic field, 10^5 gauss.

Figure 2. - Continued. Existence regions for hydrogen plasma waves.



(d-4) Axial wavelength, 100 centimeters.

(d) Concluded. Magnetic field, 10^5 gauss.

Figure 2. - Concluded. Existence regions for hydrogen plasma waves.

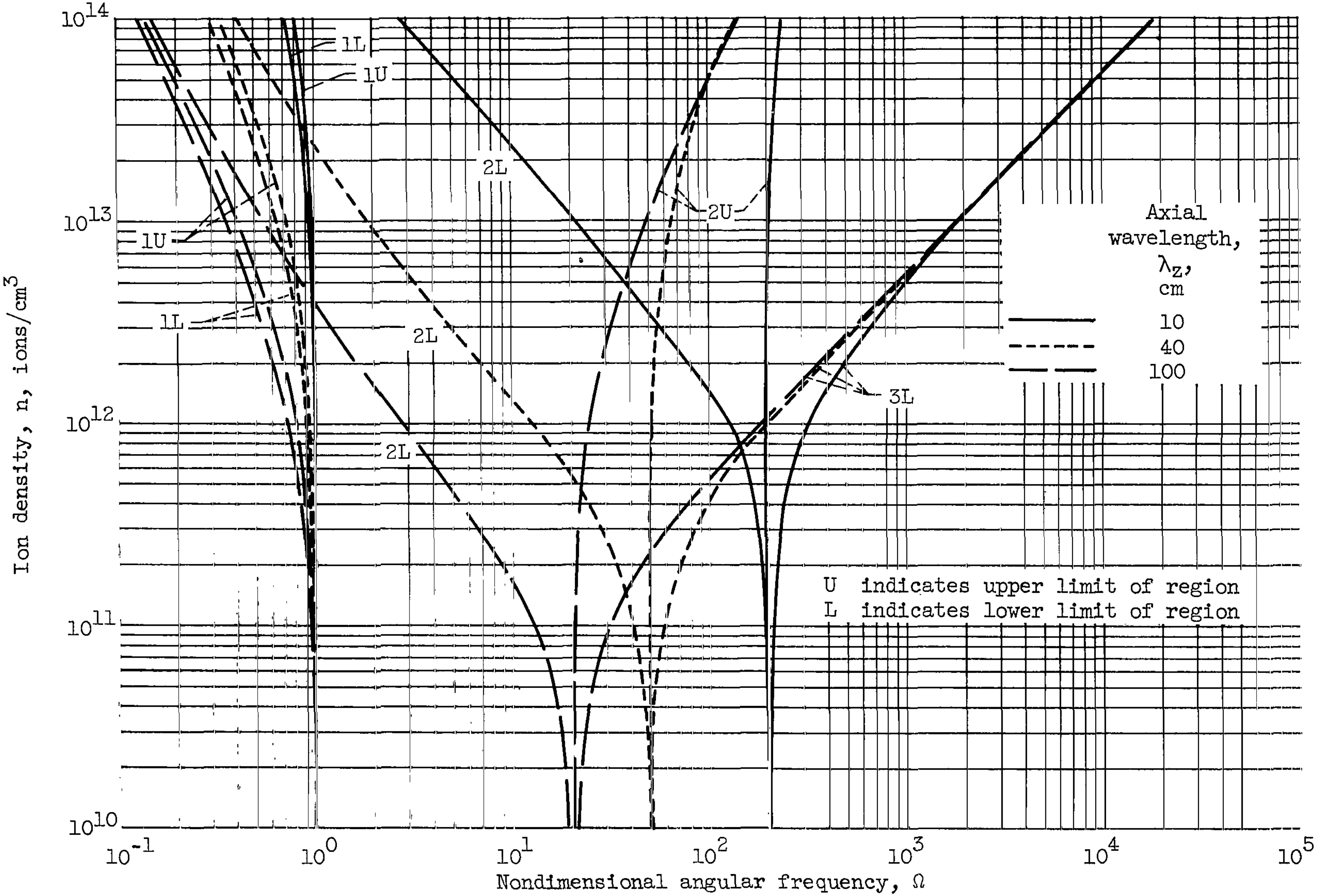


Figure 3. - Existence regions for hydrogen plasma waves with electron inertia neglected (ref. 5). Magnetic field, 10^4 gauss.

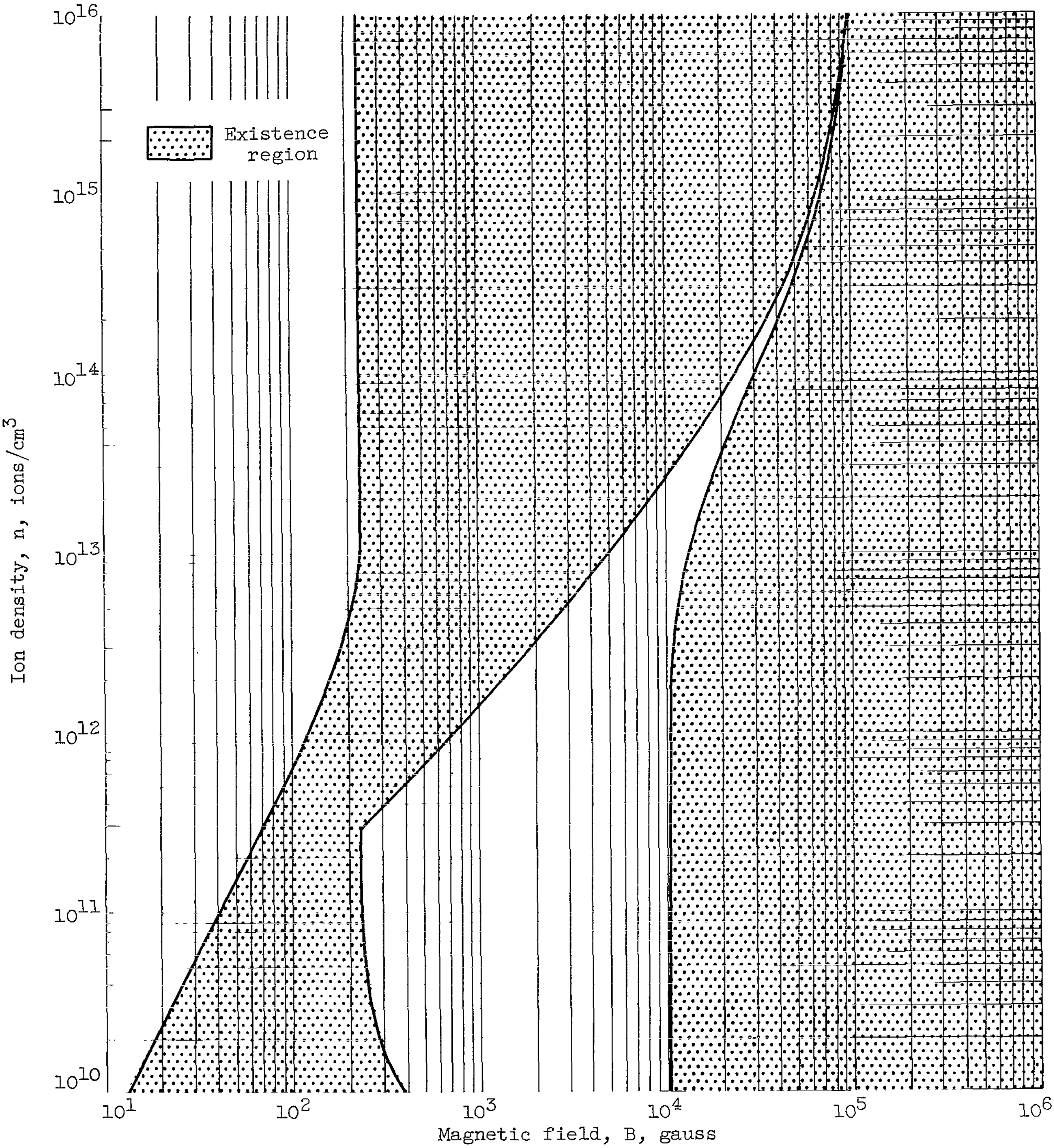


Figure 4. - Particle densities for which hydrogen plasma waves exist as function of magnetic field. Axial wavelength, 40 centimeters; ion cyclotron frequency, 9.6×10^{7} radians per second.

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